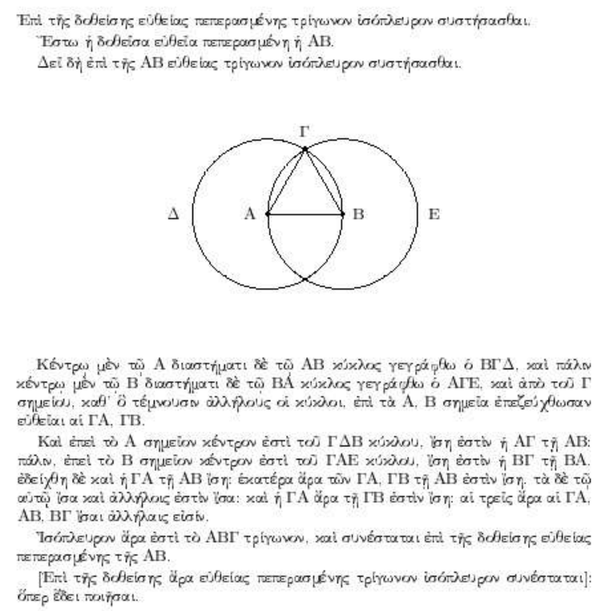
History of mathematics

The area of study known as the **history of mathematics** is primarily an investigation into the origin of discoveries in mathematics and, to a lesser extent, an investigation into the mathematical methods and notation of the past.



Ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τρίγωνον ἰσόπλευρον συστήσασθαι.
Ἔστω ἡ δοθεῖσα εὐθεῖα πεπερασμένη ἡ ΑΒ.
Δεῖ δὴ ἐπὶ τῆς ΑΒ εὐθείας τρίγωνον ἰσόπλευρον συστήσασθαι.

Κέντρῳ μὲν τῷ Α διαστήματι δὲ τῷ ΑΒ κύκλος γεγράφθω ὁ ΒΓΔ, καὶ πάλιν κέντρῳ μὲν τῷ Β διαστήματι δὲ τῷ ΒΑ κύκλος γεγράφθω ὁ ΑΓΕ, καὶ ἀπὸ τοῦ Γ σημείου, καθ' ὃ τέμνουσιν ἀλλήλους οἱ κύκλοι, ἐπὶ τὰ Α, Β σημεῖα ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓΑ, ΓΒ.
Καὶ ἐπεὶ τὸ Α σημεῖον κέντρον ἐστὶ τοῦ ΓΔΒ κύκλου, ἴση ἐστὶν ἡ ΑΓ τῇ ΑΒ· πάλιν, ἐπεὶ τὸ Β σημεῖον κέντρον ἐστὶ τοῦ ΓΑΕ κύκλου, ἴση ἐστὶν ἡ ΒΓ τῇ ΒΑ. ἐδείχθη δὲ καὶ ἡ ΓΑ τῇ ΑΒ ἴση· ἑκατέρα ἄρα τῶν ΓΑ, ΓΒ τῇ ΑΒ ἐστιν ἴση. τὰ δὲ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα· καὶ ἡ ΓΑ ἄρα τῇ ΓΒ ἐστιν ἴση· αἱ τρεῖς ἄρα αἱ ΓΑ, ΑΒ, ΒΓ ἴσαι ἀλλήλαις εἰσίν.
Ἰσόπλευρον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον, καὶ συνέσταται ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τῆς ΑΒ.
[Ἐπὶ τῆς δοθείσης ἄρα εὐθείας πεπερασμένης τρίγωνον ἰσόπλευρον συνέσταται]· ὅπερ ἔδει ποιῆσαι.

A proof from Euclid's *Elements*, widely considered the most influential textbook of all time.[1]

Before the modern age and the worldwide spread of knowledge, written examples of new mathematical developments have come to light only in a few locales. The most ancient mathematical texts available are *Plimpton 322* (Babylonian mathematics c. 1900 BC),[2] the *Rhind Mathematical Papyrus* (Egyptian mathematics c. 2000-1800 BC)[3] and the *Moscow Mathematical Papyrus* (Egyptian mathematics c. 1890 BC). All of these texts concern the so-called Pythagorean theorem, which seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry.

The Greek and Hellenistic contribution greatly refined the methods (especially through the introduction of deductive reasoning and mathematical rigor in proofs) and expanded the subject matter of mathematics.[4] The study of mathematics as a subject in its own right begins in the 6th century BC with the Pythagoreans, who coined the term "mathematics" from the ancient Greek *μάθημα* (*mathema*), meaning "subject of instruction".[5] Chinese mathematics made early contributions, including a place value system.[6] [7] The Hindu-Arabic numeral system and the rules for the use of its operations, in use throughout the world today, likely evolved over the course of the first millennium AD in India and was transmitted to the west via Islamic mathematics.[8] [9] Islamic mathematics, in turn, developed and expanded the mathematics known to these civilizations.[10] Many Greek and Arabic texts on mathematics were then translated into Latin, which led to further development of mathematics in medieval Europe.

From ancient times through the Middle Ages, bursts of mathematical creativity were often followed by centuries of stagnation. Beginning in Renaissance Italy in the 16th century, new mathematical developments, interacting with new scientific discoveries, were made at an increasing pace that continues through the present day.

Prehistoric mathematics

The Ishango bone, dating to perhaps 18,000 to 20,000 B.C.

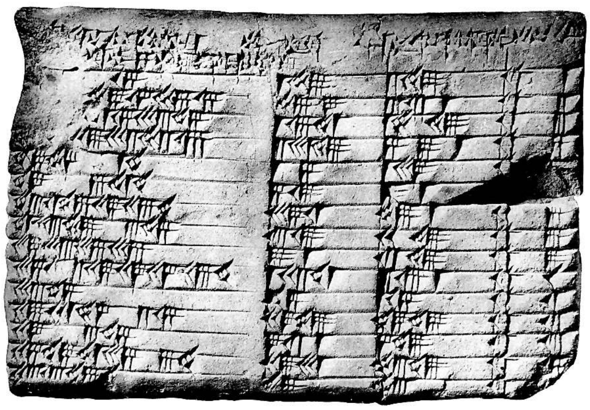
The origins of mathematical thought lie in the concepts of number, magnitude, and form.[11] Modern studies of animal cognition have shown that these concepts are not unique to humans. Such concepts would have been part of everyday life in hunter-gatherer societies. The idea of the "number" concept evolving gradually over time is supported by the existence of languages which preserve the distinction between "one", "two", and "many", but not of numbers larger than two.[11]

The oldest known possibly mathematical object is the Lebombo bone, discovered in the Lebombo mountains of Swaziland and dated to approximately 35,000 BC.[12] It consists of 29 distinct notches cut into a baboon's fibula.[13] Also prehistoric artifacts discovered in Africa and France, dated between 35,000 and 20,000 years old,[14] suggest early attempts to quantify time.[15]

The Ishango bone, found near the headwaters of the Nile river (northeastern Congo), may be as much as 20,000 years old and consists of a series of tally marks carved in three columns running the length of the bone. Common interpretations are that the Ishango bone shows either the earliest known demonstration of sequences of prime numbers[13] or a six month lunar calendar.[16] In the book *How Mathematics Happened: The First 50,000 Years*, Peter Rudman argues that the development of the concept of prime numbers could only have come about after the concept of division, which he dates to after 10,000 BC, with prime numbers probably not being understood until about 500 BC. He also writes that "no attempt has been made to explain why a tally of something should exhibit multiples of two, prime numbers between 10 and 20, and some numbers that are almost multiples of 10."[17]

Predynastic Egyptians of the 5th millennium BC pictorially represented geometric designs. It has been claimed that megalithic monuments in England and Scotland, dating from the 3rd millennium BC, incorporate geometric ideas such as circles, ellipses, and Pythagorean triples in their design.[18]

Mesopotamian mathematics



The Babylonian mathematical tablet Plimpton 322, dated to 1800 BC.

Babylonian mathematics refers to any mathematics of the people of Mesopotamia (modern Iraq) from the days of the early Sumerians through the Hellenistic period almost to the dawn of Christianity.[19] It is named Babylonian mathematics due to the central role of Babylon as a place of study. Later under the Arab Empire, Mesopotamia, especially Baghdad, once again became an important center of study for Islamic mathematics.

In contrast to the sparsity of sources in Egyptian mathematics, our knowledge of Babylonian mathematics is derived from more than 400 clay tablets unearthed since the 1850s.[20] Written in Cuneiform script, tablets were inscribed whilst the clay was moist, and baked hard in an oven or by the heat of the sun. Some of these appear to be graded homework.

The earliest evidence of written mathematics dates back to the ancient Sumerians, who built the earliest civilization in Mesopotamia. They developed a complex system of metrology from 3000 BC. From around 2500 BC onwards, the Sumerians wrote multiplication tables on clay tablets and dealt with geometrical exercises and division problems. The earliest traces of the Babylonian numerals also date back to this period.[21]

The majority of recovered clay tablets date from 1800 to 1600 BC, and cover topics which include fractions, algebra, quadratic and cubic equations, and the calculation of regular reciprocal pairs.[22] The tablets also include multiplication tables and methods for solving linear and quadratic equations. The Babylonian tablet YBC 7289 gives an approximation to $\sqrt{2}$ accurate to five decimal places.

Babylonian mathematics were written using a sexagesimal (base-60) numeral system. From this derives the modern day usage of 60 seconds in a minute, 60 minutes in an hour, and 360 (60 x 6) degrees in a circle, as well as the use of seconds and minutes of arc to denote fractions of a degree. Babylonian advances in mathematics were facilitated by the fact that 60 has many divisors. Also, unlike the Egyptians, Greeks, and Romans, the Babylonians had a true place-value system, where digits written in the left column represented larger values, much as in the decimal system. They lacked, however, an equivalent of the decimal point, and so the place value of a symbol often had to be inferred from the context.

Egyptian mathematics

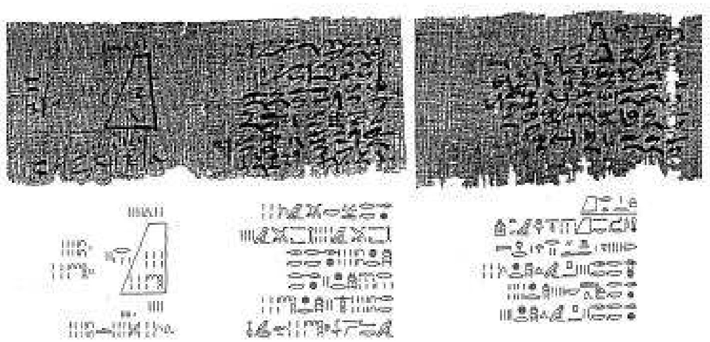


Image of Problem 14 from the Moscow Mathematical Papyrus. The problem includes a diagram indicating the dimensions of the truncated pyramid.

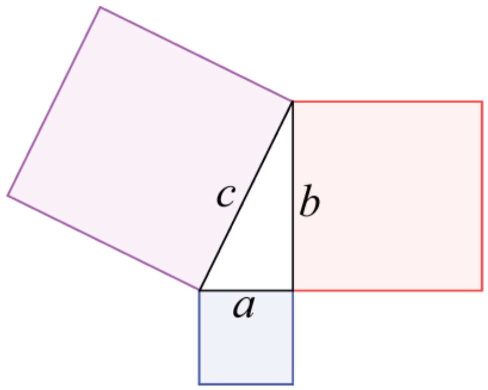
Egyptian mathematics refers to mathematics written in the Egyptian language. From the Hellenistic period, Greek replaced Egyptian as the written language of Egyptian scholars. Mathematical study in Egypt later continued under the Arab Empire as part of Islamic mathematics, when Arabic became the written language of Egyptian scholars.

The most extensive Egyptian mathematical text is the Rhind papyrus (sometimes also called the Ahmes Papyrus after its author), dated to c. 1650 BC but likely a copy of an older document from the Middle Kingdom of about 2000-1800 BC.[23] It is an instruction manual for students in arithmetic and geometry. In addition to giving area formulas and methods for multiplication, division and working with unit fractions, it also contains evidence of other mathematical knowledge,[24] including composite and prime numbers; arithmetic, geometric and harmonic means; and simplistic understandings of both the Sieve of Eratosthenes and perfect number theory (namely, that of the number 6).[25] It also shows how to solve first order linear equations[26] as well as arithmetic and geometric series.[27]

Another significant Egyptian mathematical text is the Moscow papyrus, also from the Middle Kingdom period, dated to c. 1890 BC.[28] It consists of what are today called *word problems* or *story problems*, which were apparently intended as entertainment. One problem is considered to be of particular importance because it gives a method for finding the volume of a frustum: "If you are told: A truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top. You are to square this 4, result 16. You are to double 4, result 8. You are to square 2, result 4. You are to add the 16, the 8, and the 4, result 28. You are to take one third of 6, result 2. You are to take 28 twice, result 56. See, it is 56. You will find it right."

Finally, the Berlin papyrus (c. 1300 BC[29]) shows that ancient Egyptians could solve a second-order algebraic equation.[30]

Greek Mathematics



The Pythagorean theorem. The Pythagoreans are generally credited with the first proof of the theorem.

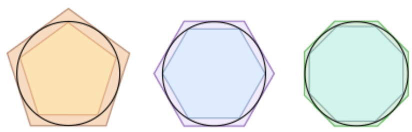
Greek mathematics refers to the mathematics written in the Greek language from the time of Thales of Miletus (~600 BC) to the closure of the Academy of Athens in 529 AD.[31] Greek mathematicians lived in cities spread over the entire Eastern Mediterranean, from Italy to North Africa, but were united by culture and language. Greek mathematics of the period following Alexander the Great is sometimes called Hellenistic mathematics.[32]

Greek mathematics was much more sophisticated than the mathematics that had been developed by earlier cultures. All surviving records of pre-Greek mathematics show the use of inductive reasoning, that is, repeated observations used to establish rules of thumb. Greek mathematicians, by contrast, used deductive reasoning. The Greeks used logic to derive conclusions from definitions and axioms, and used mathematical rigor to prove them.[33]

Greek mathematics is thought to have begun with Thales of Miletus (c. 624–c.546 BC) and Pythagoras of Samos (c. 582–c. 507 BC). Although the extent of the influence is disputed, they were probably inspired by Egyptian and Babylonian mathematics. According to legend, Pythagoras traveled to Egypt to learn mathematics, geometry, and astronomy from Egyptian priests.


One of the oldest surviving fragments of Euclid's *Elements*, found at Oxyrhynchus and dated to circa AD 100. The diagram accompanies Book II, Proposition 5.[34]

Thales used geometry to solve problems such as calculating the height of pyramids and the distance of ships from the shore. He is credited with the first use of deductive reasoning applied to geometry, by deriving four corollaries to Thales' Theorem. As a result, he has been hailed as the first true mathematician and the first known individual to whom a mathematical discovery has been attributed.[35] Pythagoras established the Pythagorean School, whose doctrine it was that mathematics ruled the universe and whose motto was "All is number".[36] It was the Pythagoreans who coined the term "mathematics", and with whom the study of mathematics for its own sake begins. The Pythagoreans are credited with the first proof of the Pythagorean theorem,[37] though the statement of the theorem has a long history, and with the proof of the existence of irrational numbers.[38] [39]


Archimedes used the method of exhaustion to approximate the value of pi.

Plato (428/427 BC – 348/347 BC) is important in the history of mathematics for inspiring and guiding others.[40] His Platonic Academy, in Athens, became the mathematical center of the world in the 4th century BC, and it was from this school that the leading mathematicians of the day, such as Eudoxus of Cnidus, came from.[41] Plato also discussed the foundations of mathematics, clarified some of the definitions (e.g. that of a line as "breadthless length"), and reorganized the assumptions.[42] The analytic method is ascribed to Plato, while a formula for obtaining Pythagorean triples bears his name.[41]

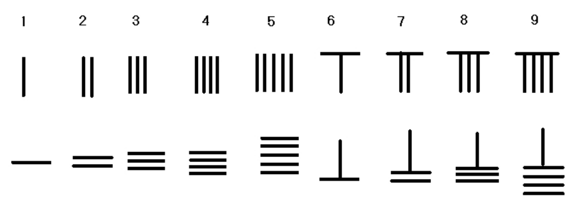
Eudoxus (408–c.355 BC) developed the method of exhaustion, a precursor of modern integration[43] and a theory of ratios that avoided the problem of incommensurable magnitudes.[44] The former allowed the calculations of areas and volumes of curvilinear figures,[45] while the latter enabled subsequent geometers to make significant advances in geometry. Though he made no specific technical mathematical discoveries, Aristotle (384—c.322 BC) contributed significantly to the development of mathematics by laying the foundations of logic.[46]

In the 3rd century BC, the premier center of mathematical education and research was the Musaeum of Alexandria.[47] It was there that Euclid (c. 300 BC) taught, and wrote the *Elements*, widely considered the most successful and influential textbook of all time.[1] The *Elements* introduced mathematical rigor through the axiomatic method and is the earliest example of the format still used in mathematics today, that of definition, axiom, theorem, and proof. Although most of the contents of the *Elements* were already known, Euclid arranged them into a single, coherent logical framework.[48] The *Elements* was known to all educated people in the West until the middle of the 20th century and its contents are still taught in geometry classes today.[49] In addition to the familiar theorems of Euclidean geometry, the *Elements* was meant as an introductory textbook to all mathematical subjects of the time, such as number theory, algebra and solid geometry,[48] including proofs that the square root of two is irrational and that there are infinitely many prime numbers. Euclid also wrote extensively on other subjects, such as conic sections, optics, spherical geometry, and mechanics, but only half of his writings survive.[50]

Archimedes (c.287–212 BC) of Syracuse, widely considered the greatest mathematician of antiquity,[51] used the method of exhaustion to calculate the area under the arc of a parabola with the summation of an infinite series, in a manner not too dissimilar from modern calculus.[52] He also showed one could use the method of exhaustion to calculate the value of π to a degree of precision as desired, and obtained the most accurate value of π then known, $3\frac{1}{7}<\pi<\frac{10}{71}$.[53] He also studied the spiral bearing his name, obtained formulas for the volumes of surfaces of revolution (paraboloid, ellipsoid, hyperboloid),[52] and an ingenious system for expressing very large numbers.[54] While he is also known for his contributions to physics and several advanced mechanical devices, Archimedes himself placed far greater value on the products of his thought and general mathematical principles.[55] He regarded as his greatest achievement his finding of the surface area and volume of a sphere, which he obtained by proving these are 2/3 the surface area and volume a cylinder circumscribing the sphere.[56]

Chinese mathematics

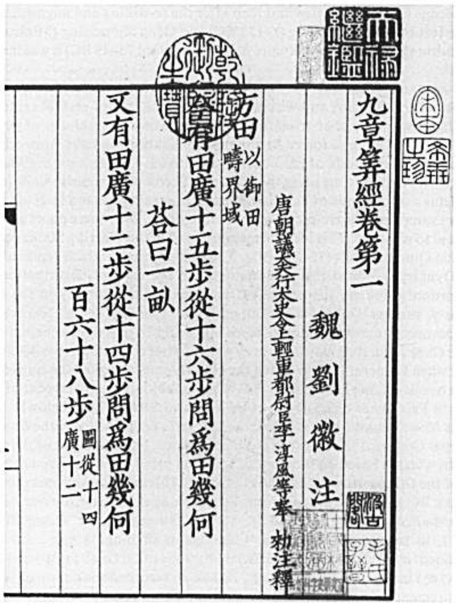
Early Chinese mathematics is so different from that of other parts of the world that it is reasonable to assume independent development.[57] The oldest extant mathematical text from China is the *Chou Pei Suan Ching*, variously dated to between 1200 BC and 100 BC, though a date of about 300 BC appears reasonable.[58]



Counting rod numerals

Of particular note is the use in Chinese mathematics of a decimal positional notation system, the so-called "rod numerals" in which distinct ciphers were used for numbers between 1 and 10, and additional ciphers for powers of ten.[59] Thus, the number 123 would be written using the symbol for "1", followed by the symbol for "100", then the symbol for "2" followed by the symbol for "10", followed by the symbol for "3". This was the most advanced number system in the world at the time, apparently in use several centuries before the common era and well before the development of the Indian numeral system.[60] Rod numerals allowed the representation of numbers as large as desired and allowed calculations to be carried out on the *suan pan*, or (Chinese abacus). The date of the invention of the *suan pan* is not certain, but the earliest written mention dates from AD 190, in Xu Yue's *Supplementary Notes on the Art of Figures*.

The oldest existent work on geometry in China comes from the philosophical Mohist canon c. 330 BC, compiled by the followers of Mozi (470–390 BC). The *Mo Jing* described various aspects of many fields associated with physical science, and provided a small number of geometrical theorems as well.[61]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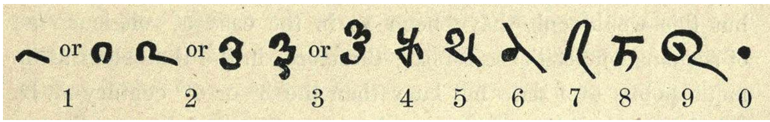
The Nine Chapters on the Mathematical Art, one of the earliest surviving mathematical texts from China (2nd century AD).

In 212 BC, the Emperor Qin Shi Huang (Shi Huang-ti) commanded all books in the Qin Empire other than officially sanctioned ones be burned. This decree was not universally obeyed, but as a consequence of this order little is known about ancient Chinese mathematics before this date. After the book burning of 212 BC, the Han dynasty (202 BC–220 AD) produced works of mathematics which presumably expanded on works that are now lost. The most important of these is *The Nine Chapters on the Mathematical Art*, the full title of which appeared by AD 179, but existed in part under other titles beforehand. It consists of 246 word problems involving agriculture, business, employment of geometry to figure height spans and dimension ratios for Chinese pagoda towers, engineering, surveying, and includes material on right triangles and values of π.[58] It created mathematical proof for the Pythagorean theorem, and a mathematical formula for Gaussian elimination. Liu Hui commented on the work in the 3rd century AD, and gave a value of π accurate to 5 decimal places.[62] Though more of a matter of computational stamina than theoretical insight, in the 5th century AD Zu Chongzhi computed the value of π to seven decimal places, which remained the most accurate value of π for almost the next 1000 years.[62] He also established a method which would later be called Cavalieri's principle to find the volume of a sphere.[63]

The high water mark of Chinese mathematics occurs in the 13th century (latter part of the Sung period), with the development of Chinese algebra. The most important text from that period is the *Precious Mirror of the Four Elements* by Chu Shih-chieh (fl. 1280-1303), dealing with the solution of simultaneous higher order algebraic equations using a method similar to Horner's method.[62] The *Precious Mirror* also contains a diagram of Pascal's triangle with coefficients of binomial expansions through the eighth power, though both appear in Chinese works as early as 1100.[64] The Chinese also made use of the complex combinatorial diagram known as the magic square and magic circles, described in ancient times and perfected by Yang Hui (AD 1238–1298).[64]

Even after European mathematics began to flourish during the Renaissance, European and Chinese mathematics were separate traditions, with significant Chinese mathematical output in decline from the 13th century onwards. Jesuit missionaries such as Matteo Ricci carried mathematical ideas back and forth between the two cultures from the 16th to 18th centuries, though at this point far more mathematical ideas were entering China than leaving.[64]

Indian mathematics



The numerals used in the Bakhshali manuscript, dated between the 2nd century BCE and the 2nd century CE.

Brahmi numerals (lower row) in India in the 1st century CE

The earliest civilization on the Indian subcontinent is the Indus Valley Civilization that flourished between 2600 and 1900 BC in the Indus river basin. Their cities were laid out with geometric regularity, but no known mathematical documents survive from this civilization.[65]

The oldest extant mathematical records from India are the *Shatapatha Brahmana* (c. 9th century BC but estimates of the date vary widely). The Sulba Sutras (c. 800 BC–200 AD),[66] appendices to religious texts which give simple rules for constructing altars of various shapes, such as squares, rectangles, parallelograms, and others.[67] The Sulba Sutras give methods for constructing a circle with approximately the same area as a given square, which imply several different approximations of the value of π,[68] [69] In addition, they compute the square root of 2 to several decimal places, list Pythagorean triples, and give a statement of the Pythagorean theorem.[70] Mesopotamian influence at this stage is considered likely.[66]

Pāṇini (c. 5th century BC) formulated the rules for Sanskrit grammar.[71] His notation was similar to modern mathematical notation, and used metarules, transformations, and recursion. Pingala (roughly 3rd-1st centuries BC) in his treatise of prosody uses a device corresponding to a binary numeral system. His discussion of the combinatorics of meters corresponds to an elementary version of the binomial theorem. Pingala's work also contains the basic ideas of Fibonacci numbers (called *mātrāmeru*).[72]

The *Surya Siddhanta* (c. 400) introduced the trigonometric functions of sine, cosine, and inverse sine, and laid down rules to determine the true motions of the luminaries, which conforms to their actual positions in the sky.[73] This work was translated into to Arabic and Latin during the Middle Ages.

In the 5th century AD, Aryabhata wrote the *Aryabhatiya*, a slim volume, written in verse, intended to supplement the rules of calculation used in astronomy and mathematical mensuration, though with no feeling for logic or deductive methodology.[74] Though about half of the entries are wrong, it is in the *Aryabhatiya* that the decimal place-value system first appears. Several centuries later, the Muslim mathematician Abu Rayhan Biruni described the *Aryabhatiya* as a "mix of common pebbles and costly crystals".[75]

In the 7th century, Brahmagupta identified the Brahmagupta theorem, Brahmagupta's identity and Brahmagupta's formula, and for the first time, in *Brahma-sphuta-siddhanta*, he lucidly explained the use of zero as both a placeholder and decimal digit, and explained the Hindu-Arabic numeral system.[76] It was from a translation of this Indian text on mathematics (c. 770) that Islamic mathematicians were introduced to this numeral system, which they adapted as Arabic numerals. Islamic scholars carried knowledge of this number system to Europe by the 12th century, and it has now displaced all older number systems throughout the world. In the 10th century, Halayudha's commentary on Pingala's work contains a study of the Fibonacci sequence and Pascal's triangle, and describes the formation of a matrix.

In the 12th century, Bhāskara II[77] lived in southern India and wrote extensively on all then known branches of mathematic. His work contains mathematical objects equivalent or approximately equivalent to infinitesimals, derivatives, the mean value theorem and the derivative of the sine function. To what extent he anticipated the invention of calculus is a controversial subject among historians of mathematics[78]

In the 14th century, Madhava of Sangamagrama, the founder of the so-called Kerala School of Mathematics, found the Madhava–Leibniz series, and, using 21 terms, computed the value of π as 3.14159265359. Madhava also found the Madhava-Gregory series to determine the arctangent, the Madhava-Newton power series to determine sine and cosine and the Taylor approximation for sine and cosine functions .[79] In the 16th century, Jyesthadeva consolidated many of the Kerala School's developments and theorems in the *Yukti-bhāṣā*.[80] However, the Kerala School did not formulate a systematic theory of differentiation and integration, nor is there any direct evidence of their results being transmitted outside Kerala.[81] [82] [83] [84] Progress in mathematics along with other fields of science stagnated in India with the establishment of Muslim rule in India.[85] [86]

Islamic mathematics

The Islamic Empire established across Persia, the Middle East, Central Asia, North Africa, Iberia, and in parts of India in the 8th century made significant contributions towards mathematics. Although most Islamic texts on mathematics were written in Arabic, most of them were not written by Arabs, since much like the status of Greek in the Hellenistic world, Arabic was used as the written language of non-Arab scholars throughout the Islamic world at the time. Persians contributed to the world of Mathematics alongside Arabs.

Page from *The Compendious Book on Calculation by Completion and Balancing* by Muhammad ibn Mūsā al-Khwārizmī (c. AD 820)

In the 9th century, the Persian mathematician Muḥammad ibn Mūsā al-Khwārizmī wrote several important books on the Hindu-Arabic numerals and on methods for solving equations. His book *On the Calculation with Hindu Numerals*, written about 825, along with the work of Al-Kindi, were instrumental in spreading Indian mathematics and Indian numerals to the West. The word *algorithm* is derived from the Latinization of his name, Algoritmi, and the word *algebra* from the title of one of his works, *Al-Kitāb al-mukhtaṣar fī hīsāb al-ğabr wa'l-muqābala* (*The Compendious Book on Calculation by Completion and Balancing*). He gave an exhaustive explanation for the algebraic solution of quadratic equations with positive roots,[87] and he was the first to teach algebra in an elementary form and for its own sake.[88] He also discussed the fundamental method of "reduction" and "balancing", referring to the transposition of subtracted terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation. This is the operation which al-Khwārizmī originally described as *al-jabr*.[89] His algebra was also no longer concerned "with a series of problems to be resolved, but an exposition which starts with primitive terms in which the combinations must give all possible prototypes for equations, which henceforward explicitly constitute the true object of study." He also studied an equation for its own sake and "in a generic manner, insofar as it does not simply emerge in the course of solving a problem, but is specifically called on to define an infinite class of problems."[90]

Further developments in algebra were made by Al-Karaji in his treatise *al-Fakhri*, where he extends the methodology to incorporate integer powers and integer roots of unknown quantities. Something close to a proof by mathematical induction appears in a book written by Al-Karaji around 1000 AD, who used it to prove the binomial theorem, Pascal's triangle, and the sum of integral cubes.[91] The historian of mathematics, F. Woepcke,[92] praised Al-Karaji for being "the first who introduced the theory of algebraic calculus." Also in the 10th century, Abul Wafa translated the works of Diophantus into Arabic. Ibn al-Haytham was the first mathematician to derive the formula for the sum of the fourth powers, using a method that is readily generalizable for determining the general formula for the sum of any integral powers. He performed an integration in order to find the volume of a paraboloid, and was able to generalize his result for the integrals of polynomials up to the fourth degree. He thus came close to finding a general formula for the integrals of polynomials, but he was not concerned with any polynomials higher than the fourth degree.[93]

In the late 11th century, Omar Khayyam wrote *Discussions of the Difficulties in Euclid*, a book about what he perceived as flaws in Euclid's *Elements*, especially the parallel postulate. He was also the first to find the general geometric solution to cubic equations. He was also very influential in calendar reform.

In the 13th century, Nasir al-Din Tusi (Nasireddin) made advances in spherical trigonometry. He also wrote influential work on Euclid's parallel postulate. In the 15th century, Ghiyath al-Kashi computed the value of π to the 16th decimal place. Kashi also had an algorithm for calculating *n*th roots, which was a special case of the methods given many centuries later by Ruffini and Horner.

Other achievements of Muslim mathematicians during this period include the addition of the decimal point notation to the Arabic numerals, the discovery of all the modern trigonometric functions besides the sine, al-Kindi's introduction of cryptanalysis and frequency analysis, the development of analytic geometry by Ibn al-Haytham, the beginning of algebraic geometry by Omar Khayyam and the development of an algebraic notation by al-Qalasādī.[94]

During the time of the Ottoman Empire and Safavid Empire from the 15th century, the development of Islamic mathematics became stagnant.

Medieval European mathematics

Medieval European interest in mathematics was driven by concerns quite different from those of modern mathematicians. One driving element was the belief that mathematics provided the key to understanding the created order of nature, frequently justified by Plato's *Timaeus* and the biblical passage (in the *Book of Wisdom*) that God had *ordered all things in measure, and number, and weight*.[95]

Boethius provided a place for mathematics in the curriculum in the 6th century when he coined the term *quadrivium* to describe the study of arithmetic, geometry, astronomy, and music. He wrote *De institutione arithmetica*, a free translation from the Greek of Nicomachus's *Introduction to Arithmetic*; *De institutione musica*, also derived from Greek sources; and a series of excerpts from Euclid's *Elements*. His works were theoretical, rather than practical, and were the basis of mathematical study until the recovery of Greek and Arabic mathematical works.[96] [97]

In the 12th century, European scholars traveled to Spain and Sicily seeking scientific Arabic texts, including al-Khwārizmī's *The Compendious Book on Calculation by Completion and Balancing*, translated into Latin by Robert of Chester, and the complete text of Euclid's *Elements*, translated in various versions by Adelard of Bath, Herman of Carinthia, and Gerard of Cremona.[98] [99]

These new sources sparked a renewal of mathematics. Fibonacci, writing in the *Liber Abaci*, in 1202 and updated in 1254, produced the first significant mathematics in Europe since the time of Eratosthenes, a gap of more than a thousand years. The work introduced Hindu-Arabic numerals to Europe, and discussed many other mathematical problems.

The 14th century saw the development of new mathematical concepts to investigate a wide range of problems.[100] One important contribution was development of mathematics of local motion.

Thomas Bradwardine proposed that speed (V) increases in arithmetic proportion as the ratio of force (F) to resistance (R) increases in geometric proportion. Bradwardine expressed this by a series of specific examples, but although the logarithm had not yet been conceived, we can express his conclusion anachronistically by writing: V = log (F/R).[101] Bradwardine's analysis is an example of transferring a mathematical technique used by al-Kindi and Arnald of Villanova to quantify the nature of compound medicines to a different physical problem.[102]

One of the 14th-century Oxford Calculators, William Heytesbury, lacking differential calculus and the concept of limits, proposed to measure instantaneous speed "by the path that **would** be described by [a body] **if**... it were moved uniformly at the same degree of speed with which it is moved in that given instant".[103]

Heytesbury and others mathematically determined the distance covered by a body undergoing uniformly accelerated motion (today solved by integration), stating that "a moving body uniformly acquiring or losing that increment [of speed] will traverse in some given time a [distance] completely equal to that which it would traverse if it were moving continuously through the same time with the mean degree [of speed]".[104]

Nicole Oresme at the University of Paris and the Italian Giovanni di Casali independently provided graphical demonstrations of this relationship, asserting that the area under the line depicting the constant acceleration, represented the total distance traveled.[105] In a later mathematical commentary on Euclid's *Elements*, Oresme made a more detailed general analysis in which he demonstrated that a body will acquire in each successive increment of time an increment of any quality that increases as the odd numbers. Since Euclid had demonstrated the sum of the odd numbers are the square numbers, the total quality acquired by the body increases as the square of the time.[106]

Renaissance mathematics

Portrait of Luca Pacioli, a painting traditionally attributed to Jacopo de' Barbari, 1495, (Museo di Capodimonte).

During the Renaissance, the development of mathematics and of accounting were intertwined.[107] While there is no direct relationship between algebra and accounting, the teaching of the subjects and the books published often intended for the children of merchants who were sent to reckoning schools (in Flanders and Germany) or abacus schools (known as *abbaco* in Italy), where they learned the skills useful for trade and commerce. There is probably no need for algebra in performing bookkeeping operations, but for complex bartering operations or the calculation of compound interest, a basic knowledge of arithmetic was mandatory and knowledge of algebra was very useful.

Luca Pacioli's *"Summa de Arithmetica, Geometria, Proportioni et Proportionalità"* (Italian: "Review of Arithmetic, Geometry, Ratio and Proportion") was first printed and published in Venice in 1494. It included a 27-page treatise on bookkeeping, *"Particularis de Computis et Scripturis"* (Italian: "Details of Calculation and Recording"). It was written primarily for, and sold mainly to, merchants who used the book as a reference text, as a source of pleasure from the mathematical puzzles it contained, and to aid the education of their sons.[108] In *Summa Arithmetica*, Pacioli introduced symbols for plus and minus for the first time in a printed book, symbols that became standard notation in Italian Renaissance mathematics. *Summa Arithmetica* was also the first known book printed in Italy to contain algebra. It is important to note that Pacioli himself had borrowed much of the work of Piero Della Francesca whom he plagiarized.

In Italy, during the first half of the 16th century, Scipione del Ferro and Niccolò Fontana Tartaglia discovered solutions for cubic equations. Gerolamo Cardano published them in his 1545 book *Ars Magna*, together with a solution for the quartic equations, discovered by his student Lodovico Ferrari. In 1572 Rafael Bombelli published his *L'Algebra* in which he showed how to deal with the imaginary quantities that could appear in Cardano's formula for solving cubic equations.

Simon Stevin's book *De Thiende* ('the art of tenths'), first published in Dutch in 1585, contained the first systematic treatment of decimal notation, which influenced all later work on the real number system.

Driven by the demands of navigation and the growing need for accurate maps of large areas, trigonometry grew to be a major branch of mathematics. Bartholomaeus Pitiscus was the first to use the word, publishing his *Trigonometria* in 1595. Regiomontanus's table of sines and cosines was published in 1533.[109]

Mathematics during the Scientific Revolution

17th century

The 17th century saw an unprecedented explosion of mathematical and scientific ideas across Europe. Galileo observed the moons of Jupiter in orbit about that planet, using a telescope based on a toy imported from Holland. Tycho Brahe had gathered an enormous quantity of mathematical data describing the positions of the planets in the sky. Through his position as Brahe's assistant, Johannes Kepler was first exposed to and seriously interacted with the topic of planetary motion. Kepler's calculations were made simpler by the contemporaneous invention of natural logarithms by John Napier and Jost Bürgi. Kepler succeeded in formulating mathematical laws of planetary motion. The analytic geometry developed by René Descartes (1596–1650) allowed those orbits to be plotted on a graph, in Cartesian coordinates. Simon Stevin (1585) created the basis for modern decimal notation capable of describing all numbers, whether rational or irrational.

Building on earlier work by many predecessors, Isaac Newton discovered the laws of physics explaining Kepler's Laws, and brought together the concepts now known as infinitesimal calculus. Independently, Gottfried Wilhelm Leibniz developed calculus and much of the calculus notation still in use today. Science and mathematics had become an international endeavor, which would soon spread over the entire world.[110]

In addition to the application of mathematics to the studies of the heavens, applied mathematics began to expand into new areas, with the correspondence of Pierre de Fermat and Blaise Pascal. Pascal and Fermat set the groundwork for the investigations of probability theory and the corresponding rules of combinatorics in their discussions over a game of gambling. Pascal, with his wager, attempted to use the newly developing probability theory to argue for a life devoted to religion, on the grounds that even if the probability of success was small, the rewards were infinite. In some sense, this foreshadowed the development of utility theory in the 18th–19th century.

18th century

Leonhard Euler by Emanuel Handmann.

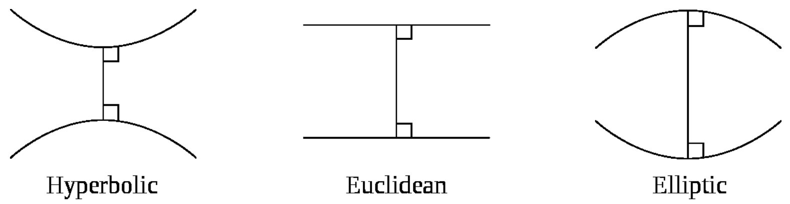
The most influential mathematician of the 18th century was arguably Leonhard Euler. His contributions range from founding the study of graph theory with the Seven Bridges of Königsberg problem to standardizing many modern mathematical terms and notations. For example, he named the square root of minus 1 with the symbol i, and he popularized the use of the Greek letter π to stand for the ratio of a circle's circumference to its diameter. He made numerous contributions to the study of topology, graph theory, calculus, combinatorics, and complex analysis, as evidenced by the multitude of theorems and notations named for him.

Other important European mathematicians of the 18th century included Joseph Louis Lagrange, who did pioneering work in number theory, algebra, differential calculus, and the calculus of variations, and Laplace who, in the age of Napoleon did important work on the foundations of celestial mechanics and on statistics.

Modern mathematics

19th century

Throughout the 19th century mathematics became increasingly abstract. In the 19th century lived Carl Friedrich Gauss (1777–1855). Leaving aside his many contributions to science, in pure mathematics he did revolutionary work on functions of complex variables, in geometry, and on the convergence of series. He gave the first satisfactory proofs of the fundamental theorem of algebra and of the quadratic reciprocity law.



Behavior of lines with a common perpendicular in each of the three types of geometry

This century saw the development of the two forms of non-Euclidean geometry, where the parallel postulate of Euclidean geometry no longer holds. The Russian mathematician Nikolai Ivanovich Lobachevsky and his rival, the Hungarian mathematician János Bolyai, independently defined and studied hyperbolic geometry, where uniqueness of parallels no longer holds. In this geometry the sum of angles in a triangle add up to less than 180°. Elliptic geometry was developed later in the 19th century by the German mathematician Bernhard Riemann; here no parallel can be found and the angles in a triangle add up to more than 180°. Riemann also developed Riemannian geometry, which unifies and vastly generalizes the three types of geometry, and he defined the concept of a manifold, which generalize the ideas of curves and surfaces.

The 19th century saw the beginning of a great deal of abstract algebra. Hermann Grassmann in Germany gave a first version of vector spaces, William Rowan Hamilton in Ireland developed noncommutative algebra. The British mathematician George Boole devised an algebra that soon evolved into what is now called Boolean algebra, in which the only numbers were 0 and 1. Boolean algebra is the starting point of mathematical logic and has important applications in computer science.

Augustin-Louis Cauchy, Bernhard Riemann, and Karl Weierstrass reformulated the calculus in a more rigorous fashion.

Also, for the first time, the limits of mathematics were explored. Niels Henrik Abel, a Norwegian, and Évariste Galois, a Frenchman, proved that there is no general algebraic method for solving polynomial equations of degree greater than four (Abel–Ruffini theorem). Other 19th century mathematicians utilized this in their proofs that straightedge and compass alone are not sufficient to trisect an arbitrary angle, to construct the side of a cube twice the volume of a given cube, nor to construct a square equal in area to a given circle. Mathematicians had vainly attempted to solve all of these problems since the time of the ancient Greeks. On the other hand, the limitation of three dimensions in geometry was surpassed in the 19th century through considerations of parameter space and hypercomplex numbers.

Abel and Galois's investigations into the solutions of various polynomial equations laid the groundwork for further developments of group theory, and the associated fields of abstract algebra. In the 20th century physicists and other scientists have seen group theory as the ideal way to study symmetry.

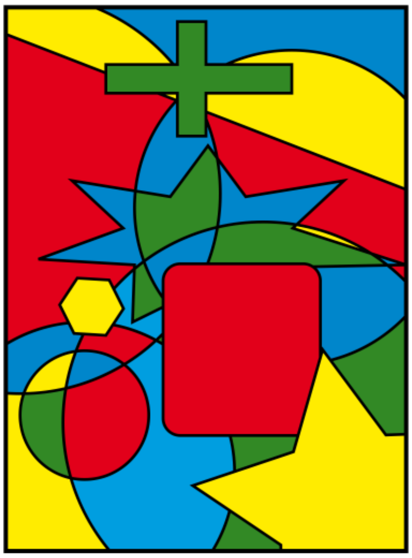
In the later 19th century, Georg Cantor established the first foundations of set theory, which enabled the rigorous treatment of the notion of infinity and has become the common language of nearly all mathematics. Cantor's set theory, and the rise of mathematical logic in the hands of Peano, L. E. J. Brouwer, David Hilbert, Bertrand Russell, and A.N. Whitehead, initiated a long running debate on the foundations of mathematics.

The 19th century saw the founding of a number of national mathematical societies: the London Mathematical Society in 1865, the Société Mathématique de France in 1872, the Circolo Mathematico di Palermo in 1884, the Edinburgh Mathematical Society in 1883, and the American Mathematical Society in 1888. The first international, special-interest society, the Quaternion Society, was formed in 1899, in the context of a vector controversy.

20th century

The 20th century saw mathematics become a major profession. Every year, thousands of new Ph.D.s in mathematics are awarded, and jobs are available in both teaching and industry.

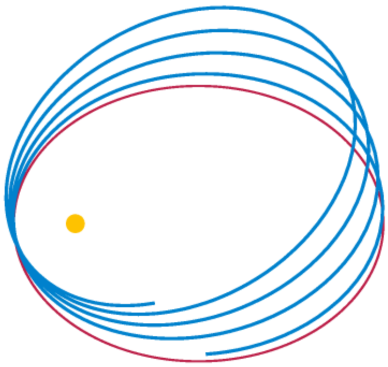
In a 1900 speech to the International Congress of Mathematicians, David Hilbert set out a list of 23 unsolved problems in mathematics. These problems, spanning many areas of mathematics, formed a central focus for much of 20th century mathematics. Today, 10 have been solved, 7 are partially solved, and 2 are still open. The remaining 4 are too loosely formulated to be stated as solved or not.



A map illustrating the Four Color Theorem

Notable historical conjectures were finally proved. In 1976, Wolfgang Haken and Kenneth Appel used a computer to prove the four color theorem. Andrew Wiles, building on the work of others, proved Fermat's Last Theorem in 1995. Paul Cohen and Kurt Gödel proved that the continuum hypothesis is independent of (could neither be proved nor disproved from) the standard axioms of set theory. In 1998 Thomas Callister Hales proved the Kepler conjecture.

Mathematical collaborations of unprecedented size and scope took place. An example is the classification of finite simple groups (also called the "enormous theorem"), whose proof between 1955 and 1983 required 500-odd journal articles by about 100 authors, and filling tens of thousands of pages. A group of French mathematicians, including Jean Dieudonné and André Weil, publishing under the pseudonym "Nicolas Bourbaki", attempted to exposit all of known mathematics as a coherent rigorous whole. The resulting several dozen volumes has had a controversial influence on mathematical education.[111]

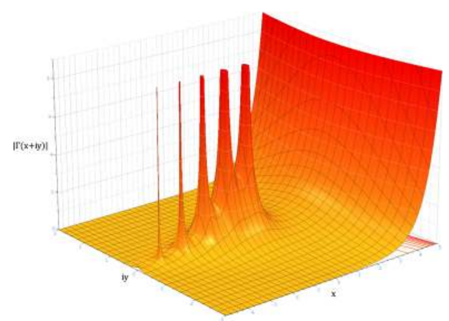


Newtonian (red) vs. Einsteinian orbit (blue) of a lone planet orbiting a star, with relativistic precession of apsides

Differential geometry came into its own when Einstein used it in general relativity. Entire new areas of mathematics such as mathematical logic, topology, and John von Neumann's game theory changed the kinds of questions that could be answered by mathematical methods. All kinds of structures were abstracted using axioms and given names like metric spaces, topological spaces etc. As mathematicians do, the concept of an abstract structure was itself abstracted and led to category theory. Grothendieck and Serre recast algebraic geometry using sheaf theory. Large advances were made in the qualitative study of dynamical systems that Poincaré had begun in the 1890s. Measure theory was developed in the late 19th and early 20th centuries. Applications of measures include the Lebesgue integral, Kolmogorov's axiomatisation of probability theory, and ergodic theory. Knot theory greatly expanded. Quantum mechanics led to the development of functional analysis. Other new areas include, Laurent Schwarz's distribution theory, fixed point theory, singularity theory and René Thom's catastrophe theory, model theory, and Mandelbrot's fractals. Lie theory with its Lie groups and Lie algebras became one of the major areas of study.

The development and continual improvement of computers, at first mechanical analog machines and then digital electronic machines, allowed industry to deal with larger and larger amounts of data to facilitate mass production and distribution and communication, and new areas of mathematics were developed to deal with this: Alan Turing's computability theory; complexity theory; Claude Shannon's information theory; signal processing; data analysis; optimization and other areas of operations research. In the preceding centuries much mathematical focus was on calculus and continuous functions, but the rise of computing and communication networks led to an increasing importance of discrete concepts and the expansion of combinatorics including graph theory. The speed and data processing abilities of computers also enabled the handling of mathematical problems that were too time-consuming to deal with by pencil and paper calculations, leading to areas such as numerical analysis and symbolic computation. Some of the most important methods and algorithms of the 20th century are: the simplex algorithm, the Fast Fourier Transform, error-correcting codes, the Kalman filter from control theory and the RSA algorithm of public-key cryptography.

At the same time, deep insights were made about the limitations to mathematics. In 1929 and 1930, it was proved the truth or falsity of all statements formulated about the natural numbers plus one of addition and multiplication, was decidable, i.e. could be determined by some algorithm. In 1931, Kurt Gödel found that this was not the case for the natural numbers plus both addition and multiplication; this system, known as Peano arithmetic, was in fact incompletable. (Peano arithmetic is adequate for a good deal of number theory, including the notion of prime number.) A consequence of Gödel's two incompleteness theorems is that in any mathematical system that includes Peano arithmetic (including all of analysis and geometry), truth necessarily outruns proof, i.e. there are true statements that cannot be proved within the system. Hence mathematics cannot be reduced to mathematical logic, and David Hilbert's dream of making all of mathematics complete and consistent needed to be reformulated.


The absolute value of the Gamma function on the complex plane.

One of the more colorful figures in 20th century mathematics was Srinivasa Aiyangar Ramanujan (1887–1920), an Indian autodidact who conjectured or proved over 3000 theorems, including properties of highly composite numbers, the partition function and its asymptotics, and mock theta functions. He also made major investigations in the areas of gamma functions, modular forms, divergent series, hypergeometric series and prime number theory.

Paul Erdős published more papers than any other mathematician in history, working with hundreds of collaborators. Mathematicians have a game equivalent to the Kevin Bacon Game, which leads to the Erdős number of a mathematician. This describes the "collaborative distance" between a person and Paul Erdős, as measured by joint authorship of mathematical papers.

As in most areas of study, the explosion of knowledge in the scientific age has led to specialization: by the end of the century there were hundreds of specialized areas in mathematics and the Mathematics Subject Classification was dozens of pages long.[112] More and more mathematical journals were published and, by the end of the century, the development of the world wide web led to online publishing.

21st century

In 2000, the Clay Mathematics Institute announced the seven Millennium Prize Problems, and in 2003 the Poincaré conjecture was solved by Grigori Perelman (who declined to accept any awards).

Most mathematical journals now have online versions as well as print versions, and many online-only journals are launched. There is an increasing drive towards open access publishing, first popularized by the arXiv.

Future of mathematics

There are many observable trends in mathematics, the most notable being that the subject is growing ever larger, computers are ever more important and powerful, the application of mathematics to bioinformatics is rapidly expanding, the volume of data to be analyzed being produced by science and industry, facilitated by computers, is explosively expanding.

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[81] (Bressoud 2002, p. 12) Quote: "There is no evidence that the Indian work on series was known beyond India, or even outside Kerala, until the nineteenth century. Gold and Pingree assert [4] that by the time these series were rediscovered in Europe, they had, for all practical purposes, been lost to India. The expansions of the sine, cosine, and arc tangent had been passed down through several generations of disciples, but they remained sterile observations for which no one could find much use."
[82] Plofker 2001, p. 293 Quote: "It is not unusual to encounter in discussions of Indian mathematics such assertions as that "the concept of differentiation was understood [in India] from the time of Manjula (... in the 10th century)" [Joseph 1991, 300], or that "we may consider Madhava to have been the founder of mathematical analysis" (Joseph 1991, 293), or that Bhaskara II may claim to be "the precursor of Newton and Leibniz in the discovery of the principle of the differential calculus" (Bag 1979, 294). ... The points of resemblance, particularly between early European calculus and the Keralese work on power series, have even inspired suggestions of a possible transmission of

mathematical ideas from the Malabar coast in or after the 15th century to the Latin scholarly world (e.g., in (Bag 1979, 285)). ... It should be borne in mind, however, that such an emphasis on the similarity of Sanskrit (or Malayalam) and Latin mathematics risks diminishing our ability fully to see and comprehend the former. To speak of the Indian “discovery of the principle of the differential calculus” somewhat obscures the fact that Indian techniques for expressing changes in the Sine by means of the Cosine or vice versa, as in the examples we have seen, remained within that specific trigonometric context. The differential “principle” was not generalized to arbitrary functions—in fact, the explicit notion of an arbitrary function, not to mention that of its derivative or an algorithm for taking the derivative, is irrelevant here"

[83] Pingree 1992, p. 562 Quote:"One example I can give you relates to the Indian Mādhava's demonstration, in about 1400 A.D., of the infinite power series of trigonometrical functions using geometrical and algebraic arguments. When this was first described in English by Charles Whish, in the 1830s, it was heralded as the Indians' discovery of the calculus. This claim and Mādhava's achievements were ignored by Western historians, presumably at first because they could not admit that an Indian discovered the calculus, but later because no one read anymore the *Transactions of the Royal Asiatic Society*, in which Whish's article was published. The matter resurfaced in the 1950s, and now we have the Sanskrit texts properly edited, and we understand the clever way that Mādhava derived the series *without* the calculus; but many historians still find it impossible to conceive of the problem and its solution in terms of anything other than the calculus and proclaim that the calculus is what Mādhava found. In this case the elegance and brilliance of Mādhava's mathematics are being distorted as they are buried under the current mathematical solution to a problem to which he discovered an alternate and powerful solution."

[84] Katz 1995, pp. 173–174 Quote:"How close did Islamic and Indian scholars come to inventing the calculus? Islamic scholars nearly developed a general formula for finding integrals of polynomials by A.D. 1000—and evidently could find such a formula for any polynomial in which they were interested. But, it appears, they were not interested in any polynomial of degree higher than four, at least in any of the material that has come down to us. Indian scholars, on the other hand, were by 1600 able to use ibn al-Haytham's sum formula for arbitrary integral powers in calculating power series for the functions in which they were interested. By the same time, they also knew how to calculate the differentials of these functions. So some of the basic ideas of calculus were known in Egypt and India many centuries before Newton. It does not appear, however, that either Islamic or Indian mathematicians saw the necessity of connecting some of the disparate ideas that we include under the name calculus. They were apparently only interested in specific cases in which these ideas were needed. ... There is no danger, therefore, that we will have to rewrite the history texts to remove the statement that Newton and Leibniz invented calculus. They were certainly the ones who were able to combine many differing ideas under the two unifying themes of the derivative and the integral, show the connection between them, and turn the calculus into the great problem-solving tool we have today."

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[86] Wickramasinghe, Nalin Chandra; Ikeda, Daisaku (1998). isbn = 9781851720613 *Space and eternal life* (http://books.google.com/books?). Journeyman Press. p. 79. isbn = 9781851720613.

[87] (Boyer 1991, "The Arabic Hegemony" p. 230) "The six cases of equations given above exhaust all possibilities for linear and quadratic equations having positive root. So systematic and exhaustive was al-Khwārizmī's exposition that his readers must have had little difficulty in mastering the solutions."

[88] Gandz and Saloman (1936), *The sources of Khwarizmi's algebra*, Osiris i, pp. 263–77: "In a sense, Khwarizmi is more entitled to be called "the father of algebra" than Diophantus because Khwarizmi is the first to teach algebra in an elementary form and for its own sake, Diophantus is primarily concerned with the theory of numbers".

[89] (Boyer 1991, "The Arabic Hegemony" p. 229) "It is not certain just what the terms *al-jabr* and *muqabalah* mean, but the usual interpretation is similar to that implied in the translation above. The word *al-jabr* presumably meant something like "restoration" or "completion" and seems to refer to the transposition of subtracted terms to the other side of an equation; the word *muqabalah* is said to refer to "reduction" or "balancing" - that is, the cancellation of like terms on opposite sides of the equation."

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[95] *Wisdom*, 11:21

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[110] Eves, Howard, An Introduction to the History of Mathematics, Saunders, 1990, ISBN 0-03-029558-0, p. 379, "...the concepts of calculus...(are) so far reaching and have exercised such an impact on the modern world that it is perhaps correct to say that without some knowledge of them a person today can scarcely claim to be well educated."
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- History of Mathematics Home Page (http://aleph0.clarku.edu/~djoyce/mathhist/) (David E. Joyce; Clark University). Articles on various topics in the history of mathematics with an extensive bibliography.
- The History of Mathematics (http://www.maths.tcd.ie/pub/HistMath/) (David R. Wilkins; Trinity College, Dublin). Collections of material on the mathematics between the 17th and 19th century.
- History of Mathematics (http://www.math.sfu.ca/histmath/) (Simon Fraser University).
- Earliest Known Uses of Some of the Words of Mathematics (http://jeff560.tripod.com/mathword.html) (Jeff Miller). Contains information on the earliest known uses of terms used in mathematics.
- Earliest Uses of Various Mathematical Symbols (http://jeff560.tripod.com/mathsym.html) (Jeff Miller). Contains information on the history of mathematical notations.
- Mathematical Words: Origins and Sources (http://www.economics.soton.ac.uk/staff/aldrich/Mathematical Words.htm) (John Aldrich, University of Southampton) Discusses the origins of the modern mathematical word stock.
- Biographies of Women Mathematicians (http://www.agnesscott.edu/lriddle/women/women.htm) (Larry Riddle; Agnes Scott College).
- Mathematicians of the African Diaspora (http://www.math.buffalo.edu/mad/) (Scott W. Williams; University at Buffalo).
- Fred Rickey's History of Mathematics Page (http://www.dean.usma.edu/math/people/rickey/hm/)
- A Bibliography of Collected Works and Correspondence of Mathematicians (http://astech.library.cornell.edu/ast/math/find/Collected-Works-of-Mathematicians.cfm) (Steven W. Rockey; Cornell University Library).

Organizations

- International Commission for the History of Mathematics (http://www.unizar.es/ichm/)

Journals

- Convergence (http://mathdl.maa.org/convergence/1/), the Mathematical Association of America's online Math History Magazine

Directories

- Links to Web Sites on the History of Mathematics (http://www.dcs.warwick.ac.uk/bshm/resources.html) (The British Society for the History of Mathematics)
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